THE RELATIONSHIP OF WORK AND SPEED TO THE HEAT-PRODUCTION ASSOCIATED WITH MUSCULAR ACTIVITY IN MAN, AND THEIR INFLUENCE ON EFFICIENCY

by

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Graphs are given at the end of the Summary -

Charts 1 to 6

In calculations of the efficiency of muscular work in Man, the relationship of work and speed to the heat-production are of fundamental importance, yet there is no common agreement between the views expressed by the many workers on this subject, and further these factors also have been found to influence the efficiency in exactly opposite ways. Usually either the 'gross-efficiency' or the 'net-efficiency', described by Benedict and Cathcart[1] have been made use of for this purpose. The former being understood to mean the ratio of the work done to the total heat-production and the latter the ratio of the work done to the total heat-production after subtraction of the resting heat. These Investigators made a very extensive study of the subject and found both the gross and net efficiencies tend to increase as the work was made greater: although they noted certain exceptions which were left unexplained.[2] Speed on the other hand caused an effect in the opposite direction, the net-efficiency fell as the speed was increased.[3] Recently Crowden[4] has given a table compiled from the work of Dickinson in which he

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(1) Benedict and Cathcart, "Muscular Work", Carnegie Institute of Washington 187, p.113, 1913.

(2) Ibid p.125.

(3) Ibid p.141.

(4) Crowden, J. Physiol. 80, p.395. 1934.

finds the net-efficiency remains relatively constant between the speeds of 25 and 48 pedal revolutions a minute, but outside these limits it diminished markedly in value. Dickinson[1] finds that the efficiency is low at both high and low rates of pedalling, but that it passes through a maximal value of about 21.5% when the time for one foot movement is about 0.9 second, corresponding to a speed of about 33 pedal revolutions a minute. In determinations of the effect of load upon the efficiency the Author selects the speed at which the efficiency was previously found maximal, and at this speed she found that variations in load are without appreciable effect on the efficiency.

With reference to the influence of variations of speed and variations of load upon efficiency, Garry and Wishart's[2] results are especially interesting, since in some of their experiments, results were obtained upon a bicycle provided with "a speed governor which, when set at a predetermined speed, automatically controls the subject's rate of pedalling by overloading him when he tends to pedal too fast, and releases some of the load when his rate falls. In this case the load fluctuates and is recorded graphically by the vertical movement of a writing point on a smoked drum". These Authors

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(1) J. Physiol. 67, p.247, S. DICKINSON, 1929.

(2) Garry and Wishart, J. Physiol. 72, 425, 1931.

found an optimal value for the efficiency when the speed was at the rate of 52 pedal revolutions a minute. These Authors as well as Crowden and Dickinson are of the opinion that they have confirmed the view already expressed by A. V. Hill, who in 1922, when working with his 'inertia wheel', described a certain optimal speed of movement below which the efficiency fell slowly and above which it fell rapidly.

In contrast to the opinions just quoted, it was pointed out(1) in 1923 that Benedict and Cathcart carried out an extensive series of experiments on their 'Subject' M.A.M. at speeds varying from 70 to 130 revolutions per minute. In these they found the "gross efficiency" greater at the slower rates of cycling; they did not find however a true maximal efficiency. In the same place it was pointed out that the minimal speed employed was by no means a low one, and therefore in comparison with the statements of Dickinson, and Garry and Wishart, to which reference has been made above, this was considerably higher than the levels at which the latter found their maximal value. In the same communication(1), a brief account was given of a series of measurements of the heat-production in which work was done against a brake on a stationary bicycle. In these although the speed varied from 40 to

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(1) F. A. Duffield and J. S. Macdonald, Proc. Physiol. Soc. 58, 1923, p.XIII.

85 revolutions per minute, no indication whatever of a maximal efficiency was found.

At the present moment, therefore, there are two opposing conclusions, one being that the efficiency progressively falls in value as the speed is increased; and the other is that the efficiency rises to reach a maximal value before falling. The latter appears to be confirmed by Dickinson, and by Garry and Wishart; however in the experiments of these Investigators, the load appears to have been allowed to vary simultaneously as changes were made in the speed, and this it is believed is the real cause of the discrepancy.

At the commencement of a description of the experimental procedure in carrying out these investigations, I wish to express my indebtedness to Professor J. S. Macdonald, F.R.S., for suggesting the subject in the first instance, and for his advice and interest in the work. However, in regard to the experimental work, and the treatment of the results and their conclusions, I must accept full responsibility.

The heat-production was calculated from the respiratory exchange of the experimental subject pedalling on a stationary bicycle against a brake applied to the hind wheel. The rim of the wheel, which was 201 c.m. in circumference, was loaded with lead weighing

18.6 Kg. The groove round the rim was lined with smooth metal and running in this was a modified rope-brake: the actual material employed consisted of a folded band of tape which was attached above to a spring-balance capable of weighing up to two pounds and graduated to quarter ounces. The lower end of the tape supported a weight-holder on which the load could be varied at will. In order to obtain a particular level of work-done on the brake, preliminary trials were carried out to determine the precise weight to be put on the holder.

The weights selected were determined by trial with the subject pedalling at the particular speed. From these data the value of the brake horse-power was obtained from the formula :-

$$\frac{2 \pi r n (T_1 - T_2)}{33000}$$ where T_1 is the weight on the holder, T_2 = the weight read off on the spring balance, n = number of revolutions of back wheel and r the radius.

At the commencement of pedalling, the friction registered on the spring-balance is rather greater than that shewn a few minutes later, when the moving parts have become warmed up, and to obviate any alteration in the value of the brake from this cause small quarter ounce weights were taken off the weight holder so as to maintain a constant reading on the spring-balance.

The rope brake was constructed from the description given by J. A. Ewing, F.R.S.[1] in his text-book on the steam engine, but I am indebted to Professor G. E. Scholes of the Engineering Department of this University for his advice and suggestions in this connection.

Throughout this series of experiments precisely identical routine procedure was adopted. Having chosen the speed beforehand, the requisite weights were put on the weight-holder and the subject sat in position on the saddle. A Siebe-gorman valve - previously tested for leaks - was secured in position and connected to a multiple Douglas-bag apparatus[2] by which three 100

(1) J. A. Ewing, F.R.S. Text-book "The Steam Engine and other engines", Third edition, 1910, p.291.

(2) Campbell, Douglas and Hobson, Phil. Trans. Roy. Soc. B. 210, 1921, p.3.

litre bag-fuls of expired air were collected. The time was noted and for the first five minutes he sat perfectly still on the saddle; then at a given signal he commenced work, pedalling to the beat of a metronome. Samples of expired air were collected at precisely 18, 23 and 28 minutes after the cycling commenced and the bag-filling time was carefully measured by a stop-watch actuated by a foot-pedal, in order to leave both hands free for manipulation of the taps on the Douglas-bags. Throughout this procedure readings of the spring-balance were noted each minute and the weights on the weight-holder re-adjusted as required to maintain a steady level of work.

At the completion of each experiment the metronome speed was carefully checked by timing against a stop-watch and an average of four separate one-hundred counts taken. Samples were removed from each bag and subjected to analysis by the large Haldane apparatus, actuated by the mechanical device described already by myself.[1] The expired air volumes were measured by a porcelain wet meter manufactured by Messrs. Alexander, Wright & Son, Westminster.

The experimental data, tabulated below, are arranged in six columns. In the first column the

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(1) F. A. Duffield, Proc. Physiol. Soc. 61, 1926.

number of the experiment is given - the letters A, B and C representing the three samples of expired air collected at five minutes intervals. The second column represents the speeds expressed in pedal revolutions per minute at which the experiments were made. The third and fourth columns contain the carbon dioxide and the oxygen values respectively expressed in litres per minute. In the fifth and sixth columns are given the respiratory quotient and the work-done on the brake respectively, the latter being expressed as horse-power. No special order was adopted in selecting the experiments; the 'Subject' often being given the choice of a light or heavy work-experiment on a particular day. It should be mentioned that the gaps occurring in the serial numbers of the experiments are caused by other persons also acting as subjects while the series under discussion was in progress.

TABLE I

(1) No. of Experiment	(2) Speed pedal revs.p.m.	(3) CO_2 l.p.m.	(4) O_2 l.p.m.	(5) R.Q.	(6) Work done horse-power
117 A	106.6	1.07	1.24	.865	.008
B	"	1.13	1.28	.88	"
C	"	1.11	1.25	.88	"
116 A	106.5	1.33	1.49	.89	.049
B	"	1.37	1.53	.89	"
C	"	1.29	1.48	.87	"
118 A	106.5	1.68	1.77	.95	.096
B	"	1.70	1.78	.95	"
C	"	1.65	1.81	.91	"
113 A	94.95	.944	1.03	.91	.004
B	"	.881	1.05	.84	"
C	"	.882	1.04	.84	"
112 A	94.61	1.06	1.30	.82	.045
B	"	1.15	1.26	.91	"
C	"	1.11	1.30	.85	"
111 A	95.7	1.50	1.66	.90	1.009
B	"	1.44	1.61	.89	"
C	"	1.55	1.66	.93	"

(1) No. of Experiment	(2) Speed pedal revs.p.m.	(3) CO_2 l.p.m.	(4) O_2 l.p.m.	(5) R.Q.	(6) Work done horse-power
119 A	94.5	1.85	1.92	.98	.147
B	"	1.88	2.01	.94	"
C	"	1.88	1.93	.79	"
56 A	84.28	.792	.923	.86	.0037
B	"	.797	.903	.88	"
C	"	.815	.896	.90	"
58 A	83.81	.979	1.17	.84	.047
B	"	.945	1.07	.88	"
C	"	1.01	1.18	.86	"
50 A	84.16	1.31	1.55	.84	.095
B	"	1.24	1.46	.85	"
C	"	1.31	1.55	.85	"
49 A	84.10	1.75	1.96	.89	.148
B	"	1.68	1.89	.89	
C	"	1.71	1.92	.89	
55 A	84.12	2.03	2.25	.90	.186
B	"	1.94	2.12	.91	"
C	"	2.01	2.18	.92	"

(1) No. of Experi-ment	(2) Speed pedal revs.p.m.	(3) CO_2 l.p.m.	(4) O_2 l.p.m.	(5) R.Q.	(6) Work done horse-power
68 A	40.29	.932	1.12	.82	.093
B	"	.913	1.07	.85	"
C	"	.961	1.14	.84	"
59 A	40.16	1.35	1.54	.87	.14
B	"	1.33	1.45	.91	"
C	"	1.41	1.54	.91	"
73 A	40.37	1.70	1.86	.91	.18
B	"	1.76	1.87	.93	"
C	"	1.71	1.90	.90	"
122 A	27.36	–	–	–	–
B	"	.324	.383	.84	.001
C	"	.338	.411	.82	"
121 A	26.31	.603	.680	.88	.042
B	"	.574	.666	.86	"
C	"	.585	.669	.87	"
120 A	26.43	.943	1.07	.88	.094
B	"	.955	1.09	.87	"
C	"	.976	1.09	.89	"

(1) No. of Experiment	(2) Speed pedal revs.p.m.	(3) CO_2 l.p.m.	(4) O_2 l.p.m.	(5) R.Q.	(6) Work done horse-power
67 A	62.36	.553	.669	.82	.003
B	"	.48	.614	.78	"
C	"	.523	.622	.80	"
66 A	62.33	.812	.934	.87	.05
B	"	.738	.92	.80	"
C	"	.805	.971	.83	"
65 A	62.72	1.00	1.23	.81	.08
B	"	.962	1.15	.84	"
C	"	.965	1.18	.81	"
64 A	62.47	1.55	1.64	.95	.14
B	"	1.38	1.54	.89	"
C	"	1.42	1.61	.88	"
63 A	62.4	1.71	1.93	.88	.178
B	"	1.62	1.84	.88	"
C	"	1.67	1.93	.87	"
62 A	40.54	.399	.488	.82	.002
B	"	.388	.477	.80	"
C	"	.408	.479	.85	"
61 A	40.46	.691	.800	.86	.047
B	"	.677	.786	.86	"
C	"	.640	.793	.80	"

In carrying out these investigations, where two variables occur, viz. work and speed, every endeavour has been made to keep one of these nearly constant, while the other is deliberately altered; the tables shew satisfactory achievement in this respect. Most difficulty being encountered in securing an exact level of work day by day, for although the particular weights employed were found previously by trial to give the selected work-level, when the horse-power was calculated later, some little displacement of its value was found, owing to slight variations in the friction of the brake due to altered atmospheric conditions. However the variations encountered are only small and they have been adjusted in further dealing with these results by methods to be described subsequently.

In column (5) of Table I are represented the values of the respiratory quotients, placed alongside the corresponding values of the work done (in column (6)), expressed in horse-power. These shew that the respiratory quotient increases as the work-performance becomes greater. Some exceptions to this are evident, but on the whole the result appears definite. Benedict and Cathcart[1] came to the same conclusion, but their results include a greater number of

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(1) Benedict and Cathcart "Muscular Work", pp.85 and 86.

exceptions.

From the oxygen figures and the corresponding respiratory quotients, the heat-production has been calculated from Lusk's Table[1] of calorific equivalents of one litre of oxygen. The heat-production expressed in large calories is shewn in Table II.

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(1) Lusk, Science of Nutrition, p.60.

TABLE II

No. of Experiment	Speed revs.p.m.	Rate of Heat-production in Cals.p.m.	Work done Cals.p.m.
117 A	106.6	6.07	.05
B	"	6.29	"
C	"	6.16	"
116 A	106.5	7.34	.52
B	"	7.53	"
C	"	7.26	"
118 A	106.6	8.85	1.03
B	"	8.92	"
C	"	8.94	"
113 A	94.95	5.12	.045
B	"	5.10	"
C	"	5.07	"
112 A	94.61	6.28	.486
B	"	6.26	"
C	"	6.36	"
111 A	95.70	8.20	1.08
B	"	7.91	"
C	"	8.26	"

No. of Experiment	Speed revs.p.m.	Rate of Heat-production in Cals.p.m.	Work done Cals.p.m.
119 A	94.50	9.64	1.57
B	"	10.00	"
C	"	9.72	"
56 A	84.28	4.50	.04
B	"	4.43	"
C	"	4.42	"
58 A	83.81	5.66	.498
B	"	5.27	"
C	"	5.78	"
50 A	84.16	7.53	1.02
B	"	7.14	"
C	"	7.54	"
49 A	84.10	9.65	1.58
B	"	9.32	"
C	"	9.45	"
55 A	84.12	11.10	1.99
B	"	10.50	"
C	"	10.80	"

No. of Experiment	Speed revs.p.m.	Rate of Heat-production in Cals.p.m.	Work done Cals.p.m.
67 A	62.36	3.23	.029
B	"	2.93	"
C	"	2.99	"
66 A	62.33	4.56	.541
B	"	4.42	"
C	"	4.72	"
65 A	62.72	5.92	.92
B	"	5.56	"
C	"	5.70	"
64 A	62.47	8.18	1.57
B	"	7.60	"
C	"	7.88	"
63 A	62.40	9.45	1.91
B	"	9.02	"
C	"	9.43	"
62 A	40.54	2.34	.019
B	"	2.30	"
C	"	2.33	"
61 A	40.46	3.90	.50
B	"	3.83	"
C	"	3.82	"

No. of Experiment	Speed revs.p.m.	Rate of Heat-production in Cals.p.m.	Work done Cals.p.m.
68 A	40.29	5.10	1.00
B	"	5.20	"
C	"	5.53	"
59 A	40.16	7.55	1.57
B	"	7.19	"
C	"	7.63	"
73 A	40.37	9.18	1.99
B	"	9.33	"
C	"	9.38	"
122 A	27.36	-	.013
B	"	1.86	"
C	"	1.98	"
121 A	26.31	3.34	.46
B	"	3.25	"
C	"	3.27	"
120 A	26.43	5.25	1.01
B	"	5.36	"
C	"	5.38	"

The actual experimental data given in Table I shew in each experiment the three analyses of the expired air, A, B and C, and from these the heat-production (Table II) has been calculated. During the collection of the three samples, the 'Subject' was performing the same amount of work on the brake and pedalling at precisely the same speed. In most cases the resulting three measurements of the heat-production are in fair agreement with one another, but on the other hand in some instances variations have occurred. These deviations from a mean value at first caused considerable disappointment, because it was not easy to explain how they could have arisen.

In the conduction of these experiments all possible care was taken with the collection of the data, and considerable preliminary work had been carried out before this series was commenced. Yet, in spite of this, some differences appeared in the experimental results. Now, however, it is believed that a satisfactory explanation of these variations can be given, and for this purpose the heat-production and work are plotted in such a way as specially to display these differences, Chart 1 . Here the three results of each analysis have been plotted separately and from this graph it is evident that more separation of plotted points is shewn at these parts of the curves which correspond to the greatest levels of work. Where the work-done is very small, e.g. when it is solely represented by the friction in the bicycle bearings, only a little separation of the points is found; whereas at the highest levels of work two cases out of three shew considerable scattering of the points.

With reference to the 'Subject', every endeavour was made to keep the mind concentrated on maintaining a steady rate of pedalling and on avoiding unnecessary movements. Nevertheless additional movements occasionally were observed, e.g. slight arching of the wrists, or raising or lowering of the shoulders, and these occurred mainly at the higher levels of work. It seems probable, therefore, that these occasional variations in the heat-production were due to the Subject unconsciously calling into play additional muscles over and above those which are essential for rotation of the pedals. Other explanations might also be considered e.g. a varying rate of pedalling; this however did not occur, at any rate as far as this 'Subject' was concerned. The speed of rotation of the pedals was under close observation while the cycling was in progress, and never was any lack of synchronism detected in the passage of the crank over a particular mark on the floor and the tick of the metronome. With another subject small changes of speed did occasionally occur and the experiments were discontinued.

It would almost be a waste of time to discuss further possible causes for these occasional variations in the heat-production measurements when a satisfactory explanation already has been found. And since an average of the three values will be made use of later, the point is of little importance in further dealing with these data.

In this research the probable error in the analysis of gas samples, found from the examination of 25 samples of atmospheric air, was .03 $\pm$ .003, and for oxygen 20.88 $\pm$ .05. Errors in measurement of the brake-horse-power are less than .02%, those obtained from calibrations of the meter are 28.0 cubic feet $\pm$ .3, which corresponds to about 1%. Error due to the collection of samples of air by Douglas bags was found to be the greatest encountered in these measurements, but they were below 1.5%.

Errors due to friction in the bicycle bearings have been overcome, and in this connection I am again indebted to Professor G. E. Scholes for his kindness in undertaking determinations of the friction in the bicycle bearings. As a result, friction values, varying with the speed of rotation of the moving parts, have been applied as a correction in determinations of the load on the brake given already in the above tables. The magnitude of these corrections may be seen at a glance by observing the heat-production where no other work is done on the bicycle.

Averages of the values from Table II of the heat-production expressed in Calories per minute and of the work-done expressed in the same units, are now plotted in Chart 2 as ordinate and abscissa respectively. In this chart six curves are shewn, each representing the heat-production at a particular speed of movement, varying approximately from 26 to 106 pedal-revolutions a minute. Obviously these curves are incomplete at extreme speeds. The reason for this is that at the faster rates of 95 and 106 revolutions a minute, the 'Subject' considered himself unable to undertake heavier work experiments than those already employed. At the lowest speed of approximately 26 revolutions a minute, only three experiments were carried out; the 'Subject' being unable to afford further time owing to other duties.

In Chart 2 six curves are drawn, each exhibiting the relationship of the heat-production to the work-done at six different speeds varying approximately from 26 to 106 pedal-revolutions a minute. They shew (1) that as the work is increased the heat-production increases proportionately and (2) that the rate of increase of the heat-production rises as the work-performed becomes greater. On a previous occasion(1) the heat-production was stated to vary with the work-done in a straight-line fashion - that was a first approximation - and as the result of further work this relationship is more accurately expressed by a curve. The work-done was measured at the approximate levels of 0, 0.5, 1.0, 1.5 and 2.0 Calories; but for reasons already stated, it was not found practicable to obtain four or five experiments, performed on different days, at a level more precise than the figures in the table shew. Therefore in order to obtain a correction for these slight differences in work-done, values of the heat-production were read off the curves at the precise levels chosen. These are tabulated in Table III.

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(1) F. A. Duffield and J. S. Macdonald "Relationship between Speed and Efficiency" Proc. Physiol. Soc. 58, 1923 and 59, 1924.

TABLE III

No. of Experiment	Speed in revs.p.m. R	Heat production in Cals.p.m. and read off Chart (3) H	Work done expressed in Cals.p.m. W
117	106.6	6.05	0
116	106.5	7.28	0.5
118	106.0	8.90	1.0
113	94.95	5.02	0
112	94.61	6.35	0.5
111	95.70	8.00	1.0
119	94.50	9.78	1.5
56	84.28	4.31	0
58	83.81	5.70	0.5
50	84.16	7.37	1.0
49	84.10	9.12	1.5
55	84.12	10.90	2.0
67	62.36	3.00	0
66	62.33	4.42	0.5
65	62.72	5.97	1.0

No. of Experiment	Speed in revs.p.m. R	Heat production in Cals.p.m. and read off Chart (3) H	Work done expressed in Cals.p.m. W
64	62.47	7.72	1.5
63	62.40	9.67	2.0
62	40.54	2.26	0
61	40.46	3.85	0.5
68	40.29	5.37	1.0
59	40.16	7.16	1.5
73	40.37	9.30	2.0
122	27.36	1.86	0
121	26.31	3.45	0.5
120	26.43	5.20	1.0

Value of the heat-production plotted against the work done in chart (2), shew six curves each representing the heat-production as a function of the work done. Further it was found by trial that the heat could be connected with the work by a formula of the types $H = a + bW + cW^2$ where H is the heat-production, W is the work done and 'a', 'b' and 'c' are constants at each speed, and taking co-ordinates from these curves, values of the coefficients were obtained and are given below in Table IV.

TABLE IV

Speeds	Coefficients a	Coefficients b	Coefficients c	No. of experimental data
106	6.05	1.9	.96	3
95	5.0	2.5	.44	4
85	4.3	2.7	.32	5
62.5	3.0	2.7	.31	5
40	2.25	2.9	.29	5
26	1.9	3.2	.04	3

Chart 2 shews that the total heat-production is divisible into two parts, one part being concerned with the measured work done and the other having no relation to it. The heat-production related to the work done is given by the difference between the total heat-production measured vertically from any point on a particular curve and the heat-production measured from the origin of H and W to the point where the curve leaves the ordinate. So far this is in agreement with the conclusions arrived at by Macdonald in 1916,[1] when he divided the measurements of heat-production, carried out in his human calorimeter, into two fractions, relegating one of these to work and the other to movement.

With reference to the formula given above, $H = a + bW + cW^2$, the terms $(bW + CW^2)$ taken together represent that portion of the heat-production which is concerned with the work-done measured on the brake, and consequently the fraction of the total heat-production which alone should be employed - as Macdonald did[1] - in making calculations of the efficiency. 'a', on the other hand, is a term which is unconnected with the work-done and corresponds to that fraction which Macdonald associated with movement. With reference to

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(1) J. S. Macdonald, Man's Mechanical Efficiency in Work performed and the Cost of Movements Involved (treated separately), Proc. Roy. Soc., B. 89, 1916, p. 396.

the former fraction ($bW + cW^2$) the coefficient 'b' appears to vary inversely, and 'c' directly with the speed (Table IV). However it must be pointed out that 'b' and 'c' at the different speeds are not all calculated from the same number of experimental data. For purposes of comparison, therefore, in the last column of Table IV are shewn the number of experiments which were available for this purpose, and if the coefficients calculated from five experiments alone are considered, i.e., those calculated at speeds of 85, 62.5 and 40 revolutions per minute, it will be evident that there is little variation between them; while those calculated from only three experimental findings shew much greater variations. Consequently it is believed that the coefficients 'b' and 'c' are constants having average values of 2.8 and 0.31 respectively. The formula therefore becomes: $H = a + 2.8W + .31W^2$.

The term 'a' in this formula has been shewn to vary directly with the speed of movement (Table IV) - a relationship now displayed graphically in Chart 3 where 'a' is plotted against the speed (R). This curve shews that, as the speed is reduced, the value of 'a' also becomes less, and finally, when the curve cuts the ordinate and the speed becomes zero, 'a' has a value of approximately 1.6 Calories a minute. This corresponds to the average value of the measured heat-production of the 'Subject' sitting at rest on the bicycle, which amounted to 1.65 Calories per minute. The heat-production therefore corresponding to the value 'a' is divisible into two parts - (1) the heat-production measured by the curve itself, i.e., the heat-production of movement, and (2) the heat-production measured along the ordinate from the origin of 'a' and R, to the point where the curve cuts the ordinate, which is the resting heat-production.

The values of 'a' are now plotted against the square of the speed in Chart 4 and the resulting points are found to lie approximately on a straight line, which cuts the ordinate at a point having the value of 1.6 Calories per minute, i.e., at the level of the resting heat-production. Taking coordinates from this line, values are found for the constants in the equation $a = \alpha + BR^2$; ~~and~~ these are 1.65 and .00037 for α and β respectively. Therefore when the work is zero the equation is $a = 1.65 + .00037\ R^2$. This indicates that the total heat-production, from which the heat-production due to the work has been subtracted, 'a', is equal to the resting heat-production plus the heat-production of movement; in this respect therefore it differs from the arrangement described by Macdonald.[1]

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(1) Loc. Cit.

The variation of 'a' with the square of the speed is of interest since the curve in Chart 4 refers to changes in velocity, i.e., an acceleration.

The following is a summary of the formulae derived :-

$$H = a + 2.8W + .3\ W^2 \qquad (1)$$

$$a = 1.6 + .00038\ R^2 \qquad (2)$$

Hence it follows that :-

$$H = \alpha + .00038\ R^2 + 2.8\ W + .3\ W^2 \qquad (4)$$

where H = the total heat-production measured on the brake, α = the resting heat-production when sitting on the bicycle and W = work done.

Formula (4) above shews that the total heat-production is divisible into three fractions, (1) a part related to rest (α), (2) a part concerned with movement $.00038\ R^2$ and (3) a part related to the work done on the brake ($2.8\ W + .3\ W^2$).

Although the data given above were all obtained on one experimental 'Subject', pedalling against a measured amount of work on the brake, experiments were performed also on other 'Subjects', working under precisely similar conditions; and although on any one person such a complete series of experiments was not made, each gave rise to very similar results. When plotted they yielded curves resembling those drawn above, and their data were capable of expression by the same kind of formulae. These experiments have not been included here because their introduction would have involved another problem altogether.

In many important respects the data, which already have been given in detail, are in agreement with those of other Workers. Benedict and Cathcart in 1912(1) published a very extensive series of experiments performed on their Subject M.A.M. doing work on a bicycle ergometer. Employing their data I have plotted the heat-production against the work done, and the resulting curves differ but little from those given above. A few of the points fall rather far from the curves, but nevertheless they shew clear results, and moreover their two variables may be connected by the formula $H = a + bW + cW^2$. They follow a different law, however, in relation to speed, viz., $H = lHR + k$ where H is the heat-production, R the speed and l and k are constants.

Macdonald(2) in 1912 published a series of figures of the heat-production obtained from pedalling on a stationary bicycle. These however were obtained from calorimetric measurements, and work was done against an electric brake at a speed of 60 revolutions a minute. The heat-production values, when plotted against the work, gave similar curves to the ones drawn in the earlier part of this thesis, and

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(1) Benedict and Cathcart, 'Muscular Work', p. 122 et seq.

(2) Macdonald, British Assn. Reports 1912, p. 289.

in addition the two values may be connected by the formula $H = a + bW + cW^2$. On another page (288) in those Reports, values of the heat-production as compared with variations in speed are given, which I find can be connected by the formula $H = lHR + k$, and in this respect they resemble Benedict's and Cathcart's figures rather than my own.

E. Hansen(1) carried out a series of experiments on several 'Subjects' performing work on a Krogh's ergometer. The data (on I.B.) are expressed in oxygen values (c.c. per minute), which, when plotted against the speed, give curves resembling those of Macdonald and Benedict and Cathcart. The relation of oxygen consumed to the speed of pedalling is given by the formula $M = lMR + k$, where M is the oxygen in c.c. per minute, R is the speed and l and k are constants. In the same communication, but on another 'Subject',(2) Hansen gives a further set of results in which the oxygen consumption was measured while work was done on the brake and expressed in Kgm. From the oxygen values plotted against the work done I have obtained curves similar to those drawn earlier in this communication. Again the oxygen values and the work are connected by the formula $M = a + bW + cW^2$.

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(1) Skand. Archiv. f. Physiol., 51. 1927, p. 116.

(2) Ibid., p. 90.

In conclusion the data obtained from the various sources detailed above have been shewn to present common agreement in certain respects. Firstly the consumption of energy may be connected with the work done by a formula of the general type $H = a + bW + cW^2$, and in this respect my data are in agreement with them, since they follow the same law connecting heat-production and work. But while the data given by Macdonald, Benedict and Cathcart and Hansen are all in agreement in respect of the connection between heat-production and speed which are related to one another by a formula of the type $H = lHR + k$; in my data the heat-production and speed are connected by formula of the general type $H = \alpha + \beta R^2$. The difference is considered to depend on the large and heavily loaded hind-wheel of the bicycle used in my experiments, causing it to act as an efficient flywheel. In consequence of this the pedals are made to rotate more evenly especially when they occupy the highest and lowest positions.

EFFICIENCY

From the values of the heat-production and the work done against the brake, tabulated in Table III, values of the 'gross efficiency' have been calculated. These are arranged in Table V below, column I giving the speed at which the measurements were made and the efficiency $\frac{W}{H}$ is given in the remaining columns, under the headings of the different loads (W) in Calories p.m.

TABLE V

Speed	Gross efficiency $\frac{W}{H}$ W = 0.5 cals. p.m.	W = 1.0 cals. p.m.	W = 1.5 cals. p.m.	W = 2.0 cals. p.m.
106	.068	.112	-	-
95	.077	.124	.152	-
85	.087	.136	.164	.182
62.5	.112	.166	.191	.207
40	.113	.188	.209	.218
26	.142	.192	-	-

These values of the gross efficiency are now plotted against the speed in Chart (5) which shews that as the speed increases the gross efficiency falls in magnitude. In Chart (6) the gross efficiency is plotted against the work done in Calories, the curves at the different speeds shew an increase in the gross efficiency as the work is made greater. No evidence, however, is found of a maximal efficiency either with variations of speed or load.

The 'net-efficiency' is calculated from the total heat-production from which the resting heat has been subtracted. This is expressed by $\frac{W}{H-\alpha}$, where α = the resting heat-production found by measurement to average 1.65 Calories per minute. Figures representing the net-efficiency are given in Table VI below.

TABLE VI

Speed revs.p.m.	Net-efficiency $\frac{W}{H-\alpha}$			
	W = 0.5 cals.p.m.	W = 1.0 cals.p.m.	W = 1.5 cals.p.m.	W = 2.0 cals.p.m.
106	.087	.132	-	-
95	.102	.152	.176	-
85	.119	.173	.198	.212
62.5	.179	.223	.238	.246
40	.225	.266	.266	.262
26	.255	.274	-	-

This table shews a diminishing net-efficiency as the speed of movement is increased; but on the other hand a rising net-efficiency as the work is increased.

It fails however to shew any evidence of an optimal speed at which the efficiency is maximal. There is in addition no evidence of a maximal efficiency when compared with variations in load, with this possible exception at the speed of 40 revolutions per minute where there is a slight fall following the rise.

In calculating the efficiency during the performance of muscular work, it is not permissible to employ the heat-production when only the resting heat has been subtracted. Of necessity heat is produced in raising and lowering the limbs, as well as in balancing the body on the saddle, and therefore it is not rational to include heat connected with these activities when comparisons are made between the work done and the heat-production involved in the process. Macdonald referred to this subject in the Reports of the British Association in 1912,[1] where he dealt with certain difficulties encountered in estimations of the efficiency in man; one of these being the measurement of the mechanical work performed by movement of the body and limbs. Lindhard[2] 1915 stated that the lower limbs do not

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(1) British Assn. Reports 1912, p. 288.

(2) Pflüg. Archer Band 161, 333, 1915.

balance one another and therefore raising and lowering of these involves work. E. Hansen[1] divides the total work into six different groups: Work done as measured on the ergometer, work in overcoming the frictional resistance of the wheels, that connected with movement of the lower limbs, work connected with regulation of speed, increased stabilising work and finally increased circulatory and respiratory work. He believes the stabilising work diminishes with training.

Opinions do not differ as to the reality of 'internal work' or 'Extraarbeit', but rather as to the method by which due allowance for it may be made in calculations of the efficiency. Lindhard [2] in an account of experiments carried out with the aid of a Krogh's bicycle ergometer, stated that the whole of the developed mechanical energy and the quantity proportional to the work done must first be obtained; but this he considered difficult of achievement. And after commenting on the advantages of the different base-lines made use of by Benedict and Cathcart in carrying out similar work, i.e., the heat-production measured in the lying position, sitting position, position of sitting on the bicycle while the pedals are rotated by a motor and that produced when cycling on an unloaded ergometer, Lindhard

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(1) Skand. Archiv. f. Physiol. 208, p. 48.

(2) Pflugers Archiv. 1915, CLXI, S. 335 et seq.

finally comes to the conclusion that the heat-production related to the position of sitting at rest on the bicycle is the most satisfactory subtraction to make from the total heat-production in calculations of the efficiency, and in consequence leaves the subject where it was.

By employing a method previously described by Bergh, du Bois Reymond and Zuntz,(1) Hansen(2) constructs a cardboard model for use in determining the movements of the centres of gravity of the lower limbs. He finds that the centre of gravity of a single lower limb traverses a track at a speed which is not constant, and the track is not symmetrical. As a result the centre of gravity of one lower limb is not always as much lifted in a given interval of time as the other is lowered, consequently work is required for the displacement of these centres of gravity; but the force used on one pedal may not be identical with that on the other.(3) Further he observes that the magnitude of the path varies with the height of the saddle, the work being greater with a high saddle than with a low one, since the oxygen consumed is greater in the former case, the mean oxygen figures varying from 804 c.c. per minute with a low saddle to 878 c.c. per minute with a high one.(4)

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(1) Bergh, du Bois Reymond and Zuntz. Arch. f. Anat. u. Physiol. Sup. 1904, S.20.
(2) E. Hansen, Skand Archiv. f. Physiol. 1927, S.51.
(3) Ibid. p. 56.
(4) Ibid. p. 66.

These quotations from Hansen therefore have produced further evidence of the importance of making due allowance for the heat-production associated with the internal work when calculations are made of the efficiency. Hansen endeavours to obtain measurements of the consumption of energy connected with his several groups of work enumerated above. In this however he meets with only partial success, and finally falls back on the "technische Wirkungsgrad", which corresponds to the gross-efficiency described by Benedict and Cathcart, and considers this the most suitable efficiency to employ in comparison with variations of speed and of work.

Macdonald(1) has calculated the efficiency by comparison of increments of heat-production with increments of work-performance measured simultaneously and at a constant speed of pedalling. By this means the heat-production connected with movement of the limbs, etc., disappears in the subtraction leaving only that heat which is directly connected with the work, to be used in calculations of the efficiency.

From my own data, given earlier in the text, (Table III), the efficiency may be calculated on this principle and by employing the symbols made use of previously, the efficiency is represented by $\frac{W}{H-a}$, where

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(1) J. S. Macdonald, British Assoc. Rep. 1912, p. 289.

W = the measured work done on the brake, H = the total heat-production and 'a' = the heat production of movement together with the resting-heat. The values of $\frac{W}{H-a}$ at the different speeds and at the different levels of work done have been calculated and are now tabulated below.

TABLE VII

	Incremental Efficiency. $\frac{W}{H-a}$			
Speed in rev.p.m.	W = P.5 cals.p.m.	W = 1.0 cals.p.m.	W = 1.5 cals.p.m.	W = 2.0 cals.p.m.
106	.39	.34	-	-
95	.38	.34	.32	-
85	.36	.33	.31	.33
62.5	.35	.33	.31	.33
40	.31	.32	.30	.28
26	.33	.33	-	-

The first column of Table VII shews the speeds at which the experiments were performed. In the remaining columns the values of the incremental efficiency are given alongside the particular speed at which the experiments were carried out, and at the head of each of these columns the work-done is stated.

Considered as a whole the incremental efficiency tends to increase as the speed of movement becomes greater; this relationship being most marked where the work-done is least, i.e., at a value of 0.5 Calories per minute. On the other hand when compared with alterations with work-done, the efficiency tends to fall as the work is increased. It should be stated, however, that these variations are not very great, and consequently the incremental efficiency has been found to alter within only comparatively small limits from an average value of about .33, corresponding to 33% for the 'Subject' on whom these investigations were carried out.

S U M M A R Y

The heat-production calculated from the respiratory exchange is found to vary directly with the work-done, and the rate of increase of the heat-production increases as the work is made greater. This relationship is shewn graphically and it is also expressed by the formula $H = a + 2.8\ W + 0.3\ W^2$.

The heat-production therefore is divided into two parts, of which one part ($2.8\ W + 0.3\ W^2$) is related to the work-done and the other has no connection with it. This division is compared with the two fractions of the heat-production previously described by Macdonald. Agreement also has been found between the data given in the text and the published data of other workers on this subject.

The heat-production unconnected with the work, i.e. the term 'a' in the above formula, is found to vary with the speed at which the movement was made: the precise relationship being shewn graphically and also expressed by the formula $a = \alpha + \beta R^2$, where 'a' represents the heat-production unconnected with the work, 'R' is the speed, β is a coefficient of the speed which for this 'Subject' has a value of .00038 and 'α' is the resting-heat, which corresponds

in value with the measured heat-production when the Subject sits at rest on the bicycle.

It is pointed out in the text, that this relationship between heat-production and speed, was not obtained from the data published by other Investigators. This difference however is believed to be explained by the large and heavily weighted back-wheel of the bicycle used in this research which causes it to act as a fly wheel, resulting in more even rotation of the pedals.

Gross and net-efficiencies (Benedict and Cathcart), calculated from these data, shew diminishing values as the speed is increased; but increasing values as the work-done becomes greater. No evidence of a maximal efficiency was discovered.

The Incremental Efficiency (Macdonald), also obtained from these data, gives values exhibiting only small variations from a mean figure of .33. The efficiency therefore for the 'Subject' employed in this research may be given approximately as 33%.

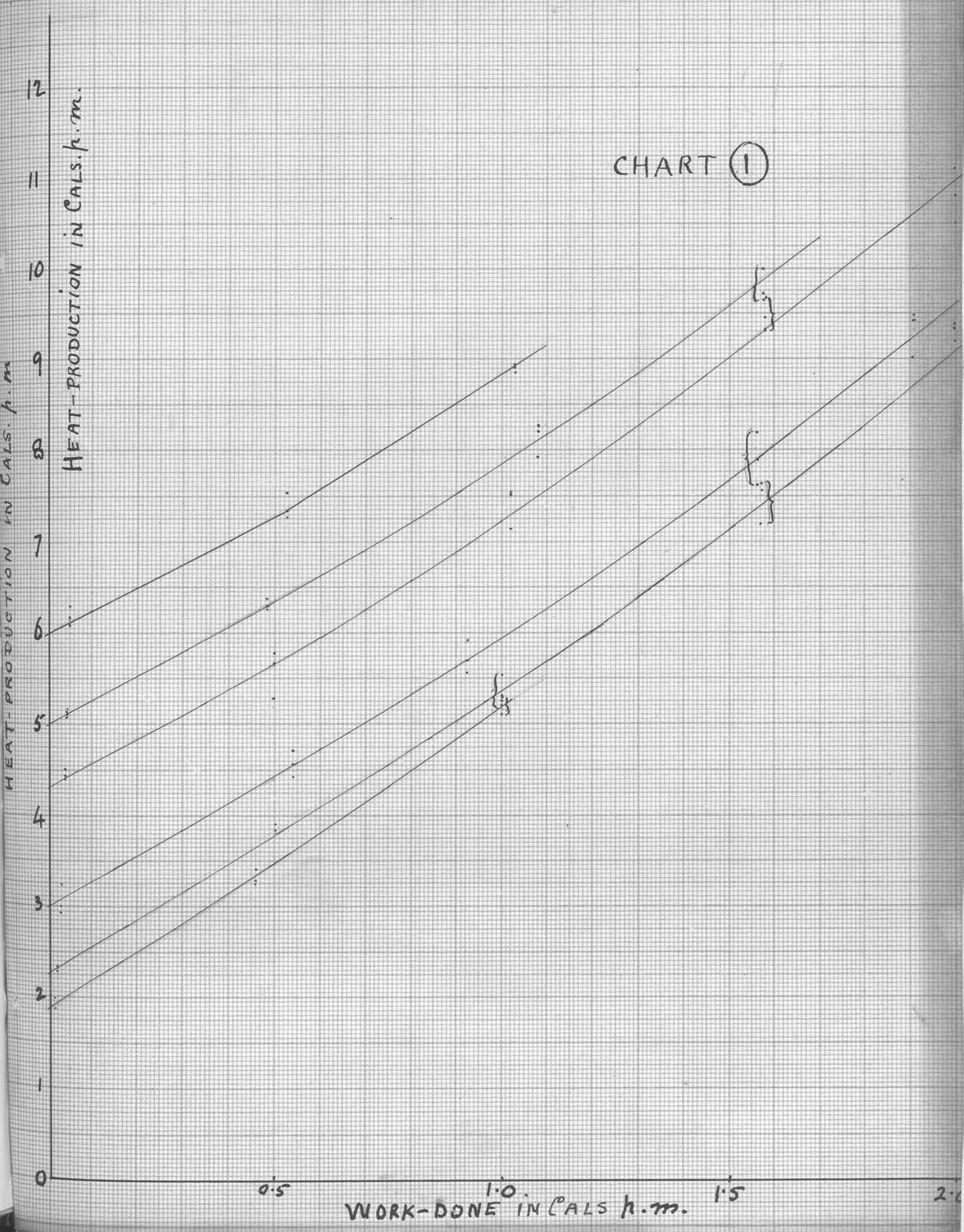


CHART (1)
HEAT-PRODUCTION IN CALS. h. m.
HEAT-PRODUCTION IN CALS. h. m
WORK-DONE IN CALS h.m.
0
1
2
3
4
5
6
7
8
9
10
11
12
0·5
1·0
1·5
2·0

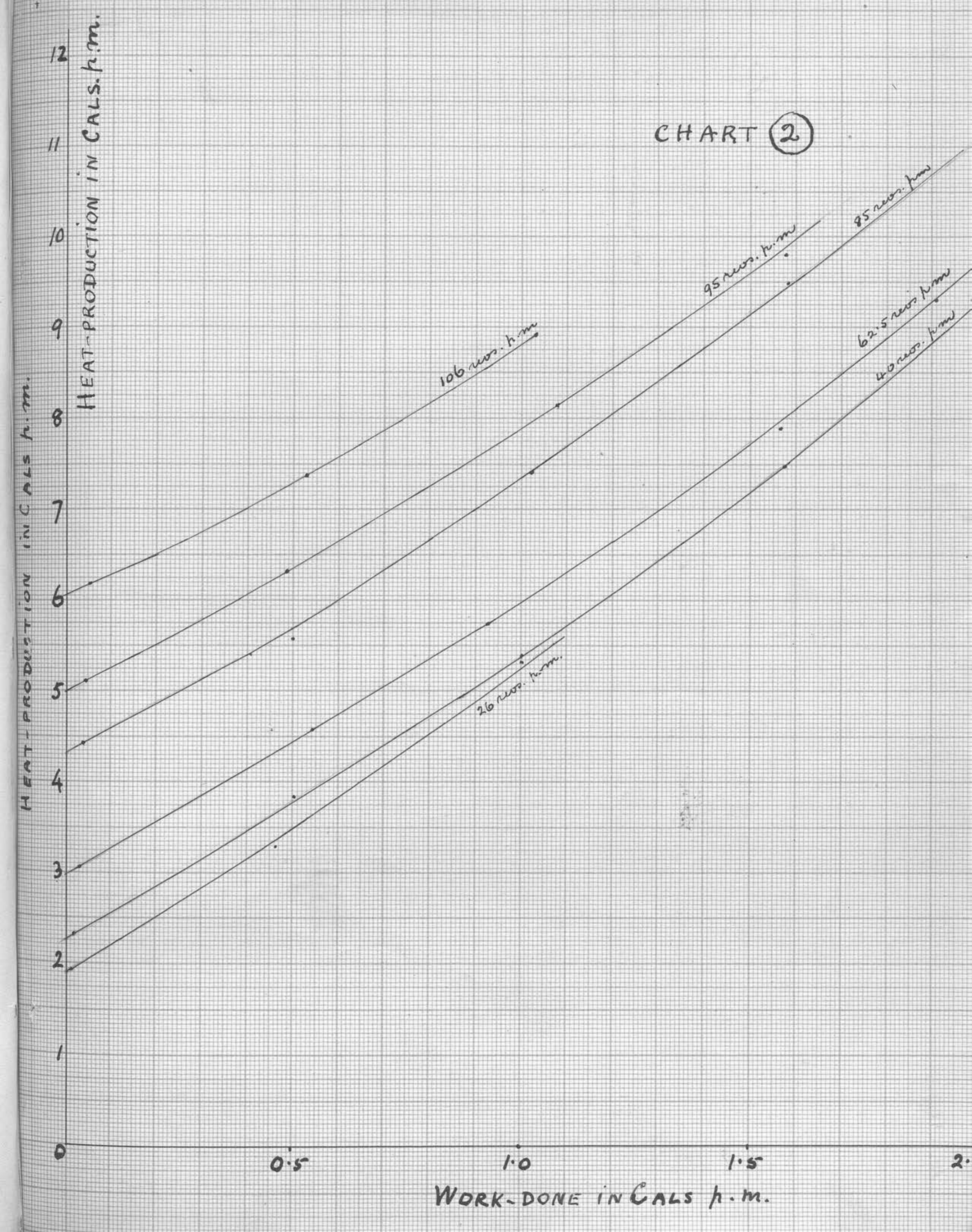


CHART (2)
HEAT-PRODUCTION IN CALS. p.m.
HEAT-PRODUCTION IN CALS p.m.
WORK-DONE IN CALS p.m.
106 revs. p.m
95 revs. p.m
85 revs. p.m
62.5 revs. p.m
40 revs. p.m
26 revs. p.m.
0
0.5
1.0
1.5
2.
1
2
3
4
5
6
7
8
9
10
11
12

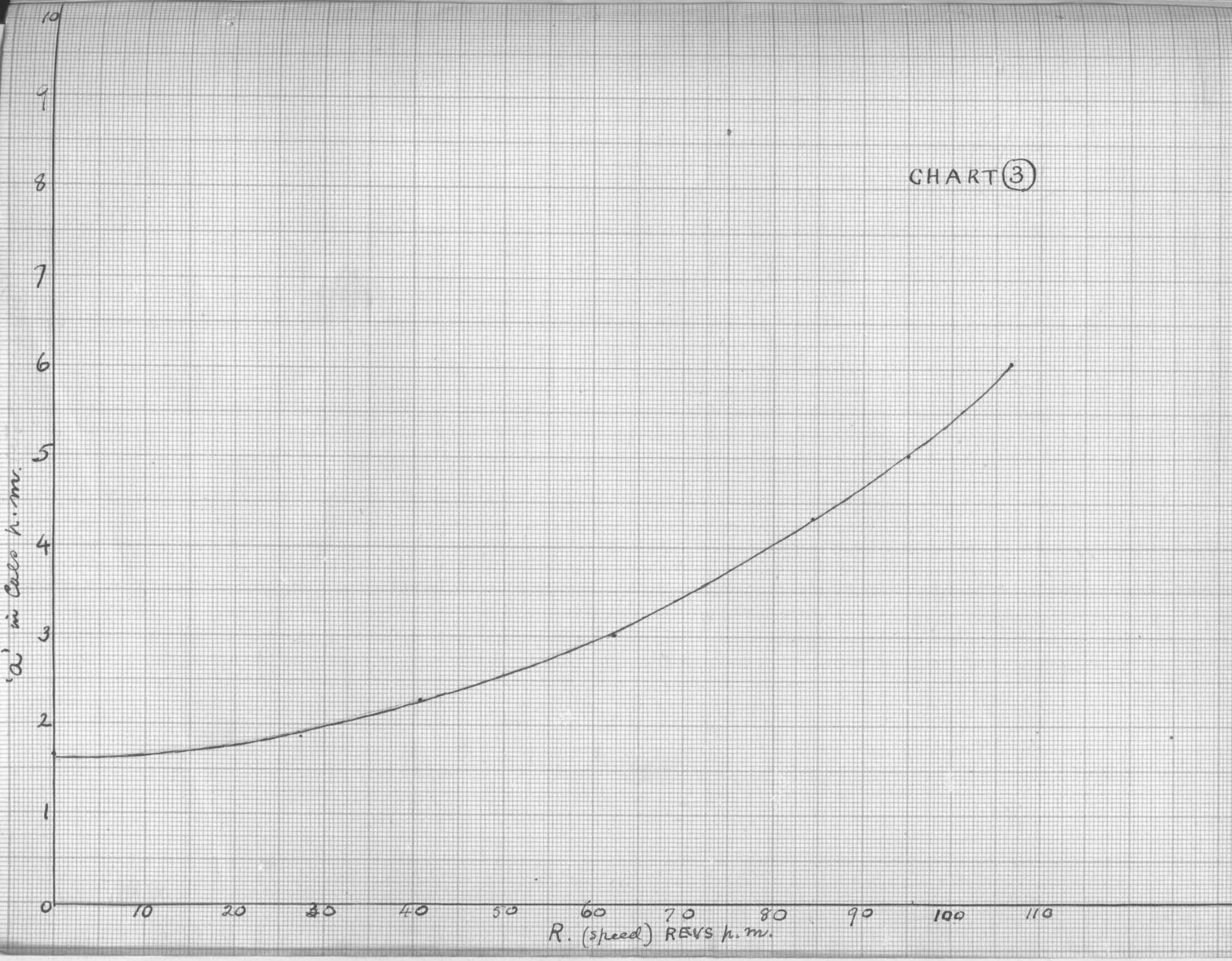
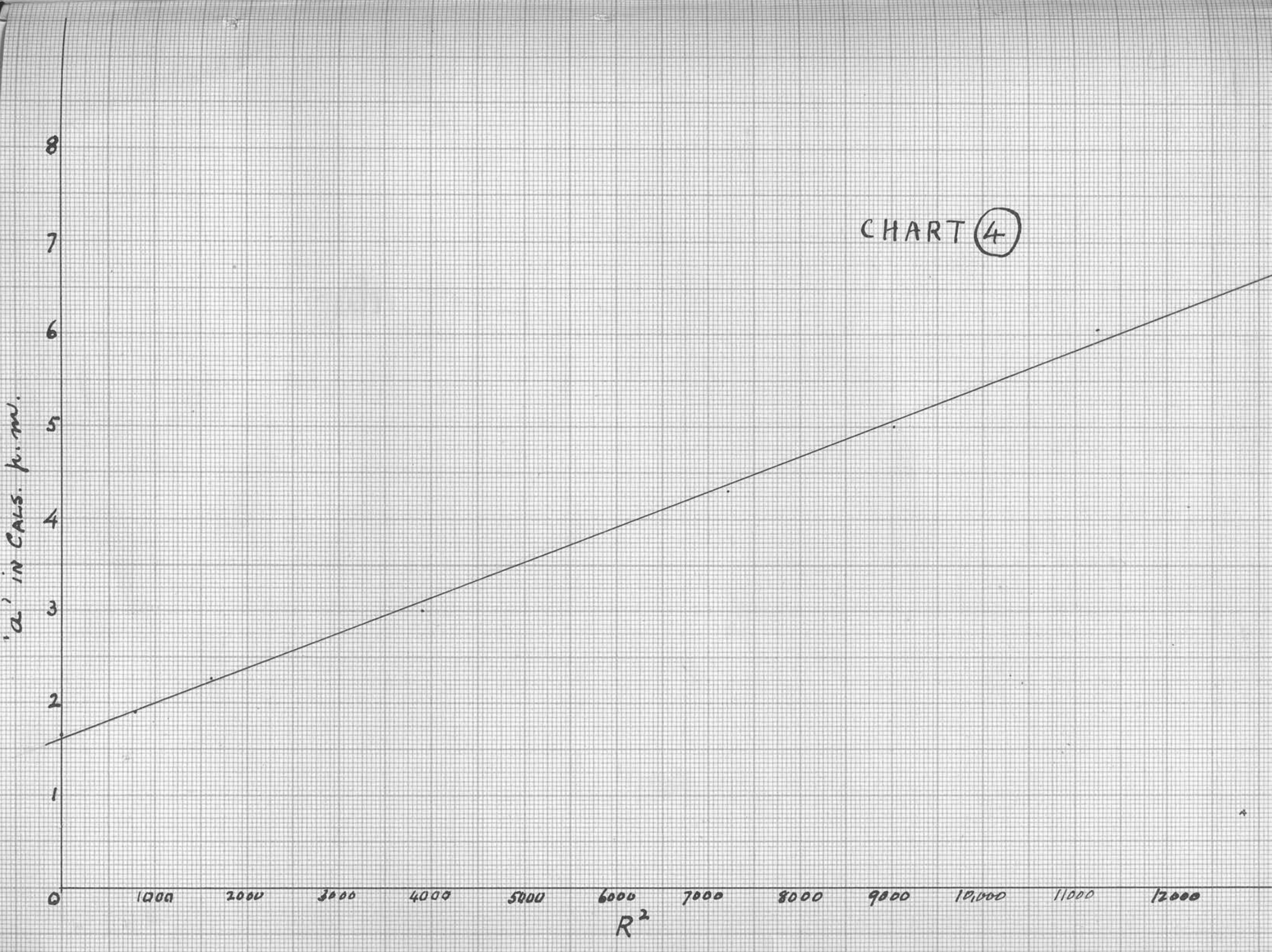


CHART 3
'Q' in Cels p.m.
R. (speed) REVS p.m.
0
1
2
3
4
5
6
7
8
9
10
10
20
30
40
50
60
70
80
90
100
110

CHART 4



'a' IN CALS. p.m.
8
7
6
5
4
3
2
1
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10,000
11000
12000
R²

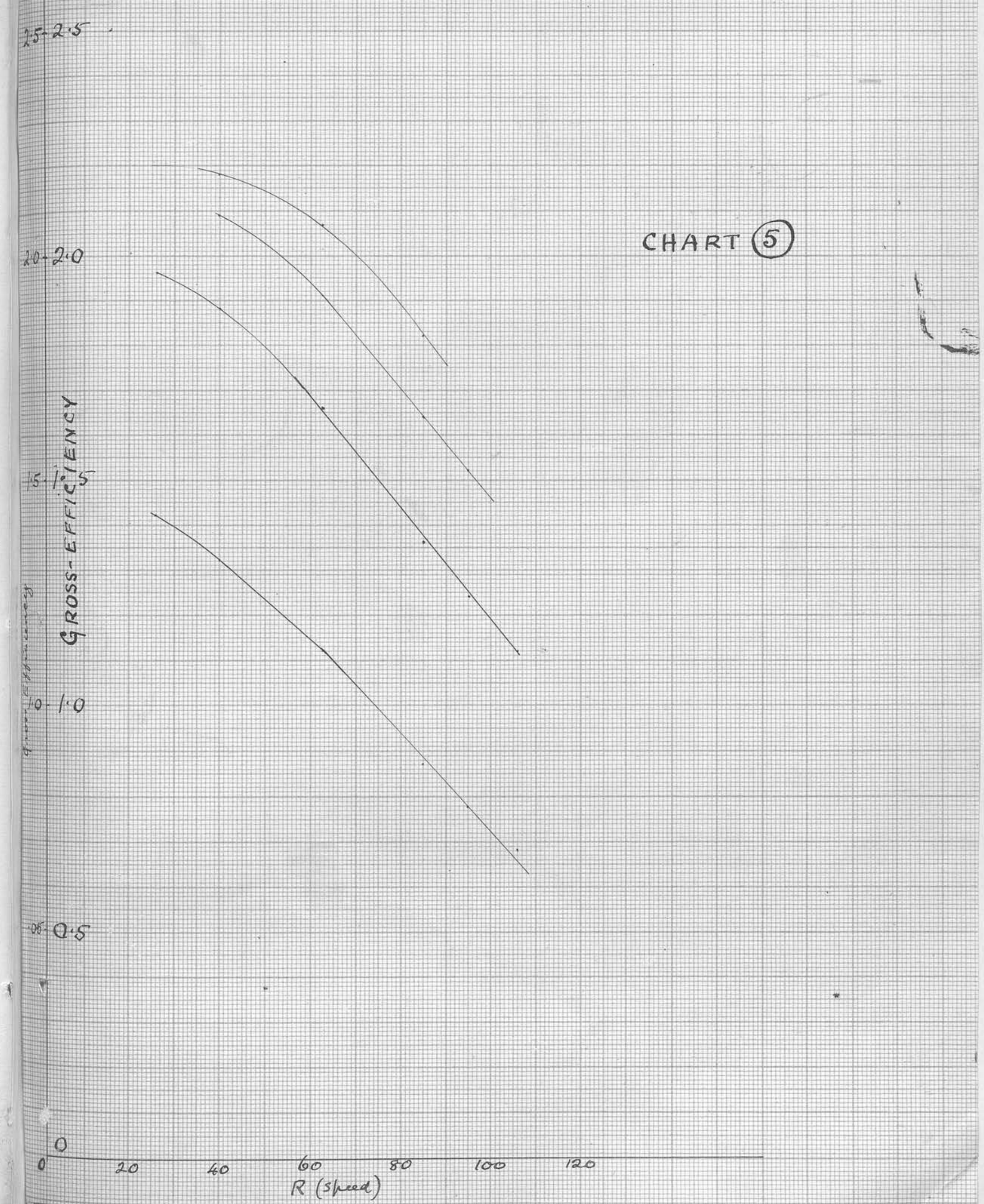


CHART 5
GROSS-EFFICIENCY
2.5
2.0
1.5
1.0
0.5
0
0
20
40
60
80
100
120
R (speed)

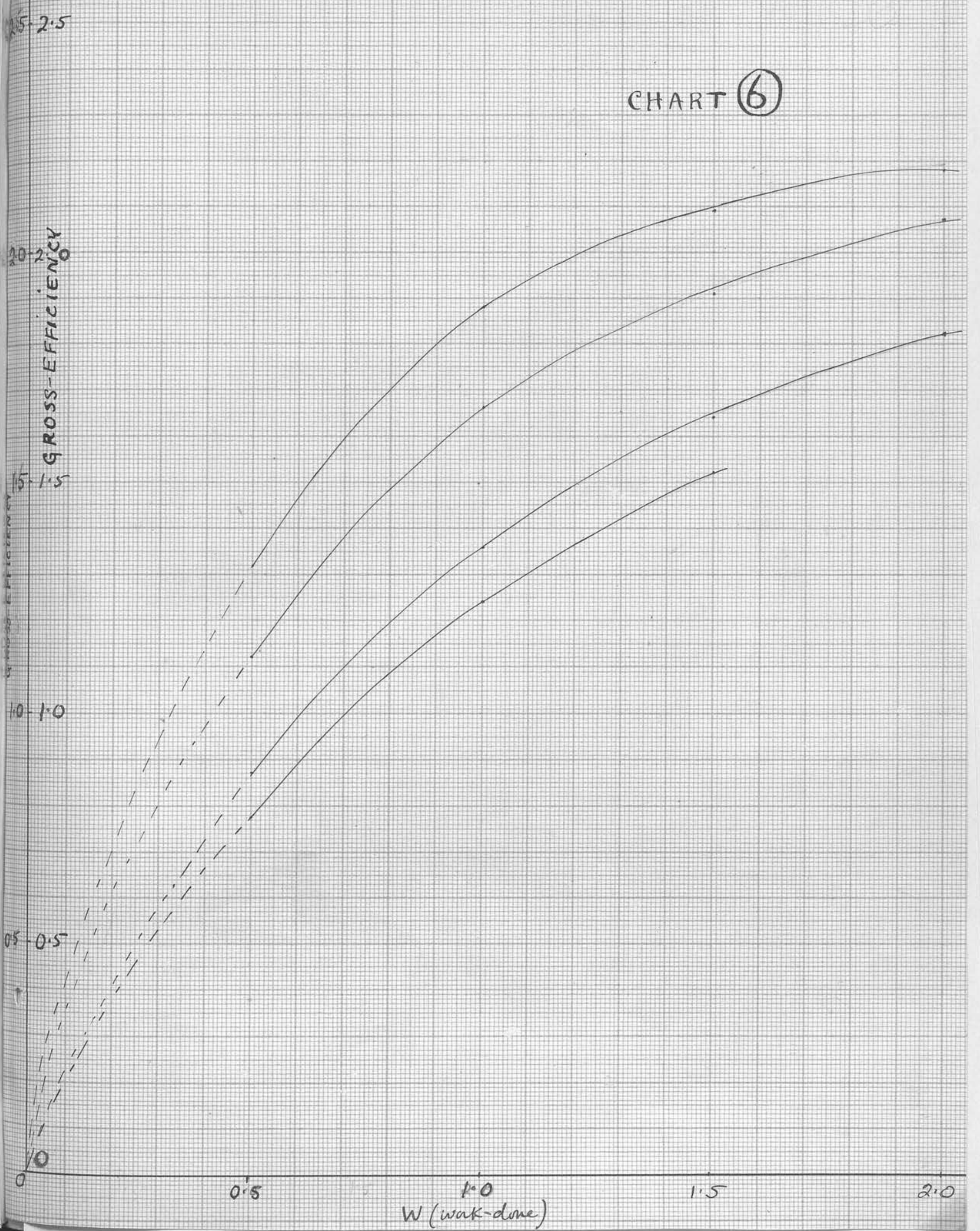


CHART 6
GROSS-EFFICIENCY
2.5
2
1.5
1.0
0.5
0
0.5
1.0
1.5
2.0
W (work-done)